

Selfacceleration with Quasidilaton

Gregory Gabadadze^{*}, Rampei Kimura[†], David Pirtskhalava[‡]

^{*}*Center for Cosmology and Particle Physics, Department of Physics,
New York University, New York, NY, 10003, USA*

[†]*Research Center for the Early Universe, The University of Tokyo, Tokyo 113-0033, Japan*

[‡]*Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126, Pisa, Italy*

Quasidilaton massive gravity is an extension of massive General Relativity to a theory with additional scale invariance and approximate internal Galilean symmetry. The theory has a novel self-accelerated solution with the metric indistinguishable (in the decoupling limit) from the de Sitter space, its curvature set by the graviton mass. The spectra of tensor, vector, and scalar perturbations on this solution contain neither ghosts, nor gradient instabilities or superluminal modes, for a range of the parameter space. This represents an example of a self-accelerated solution with viable perturbations, attainable within a low energy effective field theory.

Introduction and Summary: Both the existence and the magnitude of late-time cosmic acceleration could be hinting to a modification of gravity at cosmological distances. Of such modifications, massive gravity is arguably one of the best motivated. While the Fierz-Pauli (FP) action [1] gives the unique theory of a free spin-2 state of mass $m > 0$, generalization to a nonlinear theory has proven to be hard due to the emergence of the Boulware-Deser (BD) ghost [2]. A special set of the graviton mass and nonlinear potential terms proposed in [3, 4] (referred to as the dRGT terms) eliminates the BD ghost, as shown perturbatively to the quartic order in [4], and to all orders in the unitary gauge [5], and beyond [6]. The resulting quantum theory can be viewed as an effective field theory with the (Minkowski-space) strong coupling scale given by $\Lambda_3 = (M_{\text{Pl}} m^2)^{1/3}$ [7].

Massive gravity possesses the self-accelerated (de Sitter) vacua with curvature $\sim m^2$ [8–10]. Unfortunately, kinetic terms of either scalar or vector perturbations vanish on the selfaccelerated backgrounds [8, 11]¹. A way out might be to promote the graviton mass to a dynamical field that gives rise to a new nonlinearly realized global symmetry, as well as an approximate internal Galilean invariance [12]. This extension, dubbed quasidilaton massive gravity (QMG), retains freedom from the BD ghost. A special class of selfaccelerated solutions of QMG found in [12], suffer from a major problem: one of the perturbations of the theory necessarily flips the sign of its kinetic term on these backgrounds [13, 14]. In addition, the scale of time variation of the quasidilaton field on these special solutions is much higher than the strong coupling scale, $\sqrt{M_{\text{Pl}}} m \gg \Lambda_3$ [14], making them vulnerable to unknown high-energy physics (i.e., the special solutions cannot be obtained in the decoupling limit [12, 14]). QMG has been recently extended by De Felice and Mukohyama [15] by

derivative operators consistent with the scale symmetry; these have an effect of flipping the sign of the wrong kinetic term back to normal, hence solving the major problem. While certainly an important proof of a concept, the De Felice-Mukohyama approach does still leave the issue of UV sensitivity open [14]. The purpose of this letter is to present novel selfaccelerated solutions in QMG with viable perturbations, all attainable within the low energy field theory.

The theory and its decoupling limit: We work in the decoupling limit (DL), defined as $M_{\text{Pl}} \rightarrow \infty$, $m \rightarrow 0$, $\Lambda_3 = \text{fixed}$. The theory then captures physics at distances ranging from $\Lambda_3^{-1} \sim 1000 \text{ km}$, all the way up to (almost) $m^{-1} \sim H_0^{-1} \sim 10^{23} \text{ km}$ (see, e.g. [8, 10])².

The relevant dynamical degrees of freedom that enter into the QMG action in the DL are the helicity-2 ($h_{\mu\nu}$), helicity-1 (A_μ) and helicity-0 (π) states of the massive graviton, as well as the quasidilaton field σ . With the recent developments, a closed-form expression for the vector-scalar interactions can be obtained by using a non-dynamical antisymmetric tensor $B_{\mu\nu}$ [19, 20]. The matrix of the helicity-0 second derivatives is denoted by $\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$. The ordered index contractions on the epsilon symbols will be implied³; for example, $\varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon^{\nu_1 \nu_2 \mu_3 \mu_4} \Pi_{\nu_1}^{\mu_1} \Pi_{\nu_2}^{\mu_2} \equiv \varepsilon \varepsilon \text{IIII}$, as well as $\varepsilon_{\mu \mu_2 \mu_3 \mu_4} \varepsilon^{\nu \nu_2 \nu_3 \mu_4} \Pi_{\nu_2}^{\mu_2} \Pi_{\nu_3}^{\mu_3} \equiv \varepsilon_\mu \varepsilon^{\nu} \text{IIII}$, with an obvious generalization to terms with a different number of Π 's. The placement of indices should be treated with care for expressions involving the auxiliary tensor B_μ^ν , its square $(B^2)_\nu^\mu = B_\alpha^\mu B_\nu^\alpha$, and (the first derivative of) the vector helicity $\partial_\mu A^\nu$. For example, $\varepsilon \varepsilon B \partial A$ denotes $\varepsilon_{\mu_1 \mu_2 \mu_3 \mu_4} \varepsilon^{\nu_1 \nu_2 \mu_3 \mu_4} B_{\nu_1}^{\mu_1} \partial_{\nu_2} A^{\mu_2}$. With these conventions, and setting $\Lambda_3 = 1$, the Lagrangian describing the decoupling limit of QMG without a potential can be obtained and is written as follows:

¹ Despite of there being no reason why these kinetic terms would not be generated quantum mechanically - *e.g.* via matter loops - their vanishing at the classical level could be discouraging.

² Under certain meaningful assumptions about the UV completion, the distance scale Λ_3^{-1} can be significantly reduced for realistic

astrophysical backgrounds [16–18].

³ We use the mostly plus signature and all indices are manipulated by the flat metric $\eta_{\mu\nu}$ and its inverse. The Levi-Civita symbol, $\varepsilon_{\mu\nu\alpha\beta}$, is normalized so that $\varepsilon_{0123} = 1$.

$$\begin{aligned}
\mathcal{L}_{DL} = & -\frac{1}{2}h^{\mu\nu}(\hat{\mathcal{E}}h)_{\mu\nu} + h^{\mu\nu}\left[-\frac{1}{2}\varepsilon_\mu\varepsilon_\nu\Pi + a_2\varepsilon_\mu\varepsilon_\nu\Pi\Pi + a_3\varepsilon_\mu\varepsilon_\nu\Pi\Pi\Pi\right] + \frac{1}{M_{\text{Pl}}}h^{\mu\nu}T_{\mu\nu} - \frac{1}{4}\left[2\varepsilon\varepsilon BB + (4-8a_2)\varepsilon\varepsilon BB\Pi\right. \\
& - 4(a_2+3a_3)\varepsilon\varepsilon BB\Pi\Pi + 2\varepsilon\varepsilon B^2\Pi - 4a_2\varepsilon\varepsilon B^2\Pi\Pi - 4a_3\varepsilon\varepsilon B^2\Pi\Pi\Pi + 4\varepsilon\varepsilon B\partial A - 16a_2\varepsilon\varepsilon B\partial A\Pi - 24a_3\varepsilon\varepsilon B\partial A\Pi\Pi \\
& \left. - \omega\partial^\mu\sigma\partial_\mu\sigma + \sigma\left[\varepsilon\varepsilon\Pi - 2(a_2+1)\varepsilon\varepsilon\Pi\Pi + \frac{2}{3}(4a_2-3a_3+2)\varepsilon\varepsilon\Pi\Pi\Pi - \frac{1}{3}(2a_2-6a_3+1)\varepsilon\varepsilon\Pi\Pi\Pi\Pi\right]\right]. \quad (1)
\end{aligned}$$

Here $\hat{\mathcal{E}}$ is the Einstein operator, and $a_{2,3}$ and ω are free parameters of the theory. The π - h interactions are those of pure massive gravity [3], while the σ - π interactions are (bi-) Galileons [16]. The action is invariant under $U(1)$ gauge transformations, $A_\mu \rightarrow A_\mu + \partial_\mu\alpha$. The external stress-tensor couples to the helicity-2 field $h_{\mu\nu}$, and the latter determines geodesics of ordinary

matter. We will be interested in self-accelerated backgrounds ($T_{\mu\nu} = 0$) with the Hubble constant H ; for these $\bar{h}_{\mu\nu} = -(H^2x^2/2)\eta_{\mu\nu}$ and $\bar{\pi} = qx^2/2$ ($q = \text{const}$), while the solution for σ can in general take the form $\bar{\sigma} = (-q_0t^2 + q_1\mathbf{x}^2)/2$. We will focus on the solution with $\bar{B}^\mu_\nu = 0$, $\bar{A}_\mu = 0$. Then, the equations for the helicity-2, quasidilaton, and the helicity-0 fields give respectively

$$H^2 - 2q\left(-\frac{1}{2} + a_2q + a_3q^2\right) = 0, \quad (2)$$

$$(2a_2 - 6a_3 + 1)q^4 - 2(4a_2 - 3a_3 + 2)q^3 + 6(a_2 + 1)q^2 - 3q + \frac{\omega}{4}q_t = 0, \quad (3)$$

$$H^2\left(3a_3q^2 + 2a_2q - \frac{1}{2}\right) + q_t\left[q^3\left(\frac{2}{3}a_2 - 2a_3 + \frac{1}{3}\right) - q^2\left(2a_2 - \frac{3}{2}a_3 + 1\right) + q(a_2 + 1) - \frac{1}{4}\right] = 0, \quad (4)$$

where $q_t \equiv q_0 + 3q_1$, and q_0 and q_1 enter only via q_t , meaning that σ can be shifted by a zero mode of the d'Alembertian, $\sigma \rightarrow \sigma - \alpha t^2 + \beta\mathbf{x}^2$, $\alpha + 3\beta = 0$, without changing the background. Below we will work with q_t and q_1 ; the former is determined from the equations, while q_1 is a free 'flat direction'. Nevertheless, q_1 will affect perturbations, considerably expanding the acceptable parameter space. The equations are all linear in q_t and H^2 , therefore these can be expressed in terms of q via (4) and (2) to obtain an algebraic master equation for q from (3). The resulting equation is of the seventh order and in general not soluble analytically.

Perturbations: We start with the vector perturbations of (1). Recalling that $\partial_\mu\partial_\nu\bar{\pi} = q\eta_{\mu\nu}$ and expanding the epsilon symbols, one finds

$$\mathcal{L}_V = Q_1B_{\mu\nu}B^{\mu\nu} + Q_2B_{\mu\nu}F^{\mu\nu},$$

where $Q_2 = Q_1/(q-1) = 6a_3q^2 + 4a_2q - 1$. In the absence of the quasidilaton, the π -equation of motion obtained from (4) by setting $q_t = 0$, implies that $Q_{1,2} = 0$, leading to infinitely strong coupling of the vector perturbations [8]. In QMG however, π -equation is modified by extra terms and no longer leads to this issue: one gets dynamical and weakly coupled (in the infrared) perturbations, propagating at the speed of light. Integrating out the $B_{\mu\nu}$ field, the following necessary and sufficient

condition for the absence of vector ghosts is obtained

$$Q_1 = (q-1)(6a_3q^2 + 4a_2q - 1) > 0. \quad (5)$$

The scalar perturbations are described by the following Lagrangian

$$\begin{aligned}
\mathcal{L}_s = & A_1\delta\dot{\pi}^2 - A_2(\partial_i\delta\pi)^2 + B_1\delta\dot{\pi}\delta\dot{\sigma} - B_2\partial_i\delta\pi\partial_i\delta\sigma \\
& + C_1\delta\dot{\sigma}^2 - C_2(\partial_i\delta\sigma)^2, \quad (6)
\end{aligned}$$

where the coefficients (A, B, C) are given as follows:

$$\begin{aligned}
A_1 = & 6H^2(3a_3q + a_2) + 6c^2 \\
& + 12q_1\left[(2a_2 - 6a_3 + 1)q^2 + (3a_3 - 4a_2 - 2)q + a_2 + 1\right], \\
A_2 = & 6H^2(3a_3q + a_2) + 6c^2 + 4(q_t - q_1) \\
& \times \left[(2a_2 - 6a_3 + 1)q^2 + (3a_3 - 4a_2 - 2)q + a_2 + 1\right], \\
B_1 = B_2 = & 8(2a_2 - 6a_3 + 1)q^3 - 12(4a_2 - 3a_3 + 2)q^2 \\
& + 24(a_2 + 1)q - 6, \quad C_1 = C_2 = \omega. \quad (7)
\end{aligned}$$

For a Lorentz-invariant quasidilaton profile with $q_1 = q_0$, one gets $A_1 = A_2$ and both scalar modes propagate at the speed of light. Relaxing this condition, as we will see shortly, results in a deviation from unity of the speed of sound for only one of the two modes.

Stability and (sub)luminality: As noted above, it is impossible to obtain *the most general* closed-form analytic expressions for the viable parameter space. One can however employ certain limits and/or numerical study to show the existence of such. To this end, it is simpler to trade the free parameter a_2 for q , since a_2 can be algebraically expressed in terms of q, a_3, q_t and ω ; for any q then, one can straightforwardly find a theory with the corresponding value of a_2 . We will first consider the Lorentz-preserving case, $q_1 = q_0 = q_t/4$. Expressing H^2 (via the Friedmann equation) and a_2 (via the σ equation) in terms of the rest of the parameters, one finds a *quadratic* equation for q_t , that depends on q, a_3 and ω – the only remaining free parameters of the theory. The necessary condition for the absence of ghosts in the scalar sector is $A_1 > 0$, $C_1 > B_1^2/(4A_1)$. Let us concentrate on the limit of *large* q . Then, one can straightforwardly find the quadratic equation from which q_t can be determined,

$$-4\omega q^4 q_t^2 + 72a_3 q^7 q_t + 288a_3^2 q^{10} = 0.$$

The two solutions are⁴ $q_t = 3a_3 q^3 (3 \pm \sqrt{8\omega + 9})/\omega$. The second of these (featuring the ‘minus’ sign) does not lead to any acceptable parameter space, so we concentrate on the first one. The value of a_2 for this solution is⁵

$$a_2 = 3a_3 - \frac{1}{2} + \frac{3a_3(21 - \sqrt{8\omega + 9})}{8q} + \mathcal{O}\left(\frac{1}{q^2}\right),$$

while the (leading-order) expressions for the Hubble parameter squared and the normalization coefficient of the vector gauge kinetic term, Q_1 , are $H^2 = 2a_3 q^3$, and $Q_1 = 6a_3 q^3$. Both are positive for $a_3 q^3 > 0$, leading to dS backgrounds with ghost-free vector perturbations. With the above expressions used, the quantities determining stability of the scalar perturbations are then given by

$$A_1 = \frac{9a_3^2 q^4 (8\omega + 27 + 9\sqrt{8\omega + 9})}{2\omega},$$

$$C_1 - \frac{B_1^2}{4A_1} = \omega - \frac{2\omega(\sqrt{8\omega + 9} - 3)^2}{8\omega + 27 + 9\sqrt{8\omega + 9}}. \quad (8)$$

Both are positive for $0 < \omega < 54$. The parameter space (for $q \gg 1$) corresponding to ghost-free self-accelerated backgrounds is therefore given by

$$0 < \omega < 54, \quad \text{sgn}(a_3) = \text{sgn}(q). \quad (9)$$

All modes propagate at the speed of light, excluding any gradient instabilities⁶. This provides an analytic proof of

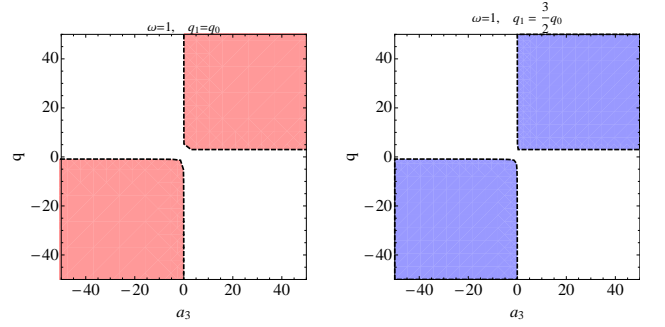


FIG. 1: Left: a subset of the parameter space for stable de Sitter solutions with a Lorentz-invariant σ profile (both scalar modes are exactly luminal). Right: the parameter space supporting stable dS backgrounds with $q_1 = 3q_0/2$. One of the scalars is strictly subluminal.

the existence of fully stable dS backgrounds for large q . The latter here is parametrically greater than unity (say, $q \sim 10$) due to the arrangement between $a_{2,3}$, while the solution itself belongs to the low energy field theory, as it is obtained in the decoupling limit. A numerical analysis shows the existence of stable self-accelerated vacua for intermediate values of q as well. One representative subset of the allowed parameter space is given in the left panel of Fig.1 (note that the allowed space does *not* extend to $|q| \sim 0$; instabilities appear for $|q| \ll 1$). On these plots we used $\omega = 1$. The colored region corresponds to one corner of the full parameter space for which the conditions of absence of the ghosts, gradient instabilities and superluminal propagation are all satisfied.

We have also explicitly checked the case $q_1 \neq q_0$; the stable de Sitter vacua are retained: the two speeds of sound of the scalar modes in this case are $c_s^2 = (x \pm \sqrt{x^2 - 4yz})/2y$, where $x = 4A_1C_2 + 4A_2C_1 - 2B_1B_2$, $y = 4A_1C_1 - B_1^2$, and $z = 4A_2C_2 - B_2^2$. Using the expressions in (7), we conclude that one of the scalar modes always propagates at the speed of light, while there is a parameter space for which the second mode is subluminal: for instance, setting $q_1 = 3q_0/2$ results in a broad parameter range consistent with the “no ghosts” and “no gradient instability” conditions, with one of the scalars strictly subluminal (the other one being luminal). A representative part of the latter parameter space is shown in the right panel of Fig.1.

Discussion and outlook: The obtained solutions should have their counterparts in the full QMG theory, beyond the decoupling limit. From our results it follows that the full QMG solutions should not have any ghost instabilities, and *if* there is any tachyon instability, it can only be very slow, with the time scale $m^{-1} \sim H_0^{-1} \sim 14 \times 10^9$ years. The full QMG solutions should turn into those of pure massive gravity as ω is taken to infinity. While the DL solutions discussed here

⁴ We have assumed $a_3 q^3 > 0$, since this has to be the case for a positive H^2 , as shown shortly.

⁵ One has to retain a subleading piece for a_2 due to a cancellation of leading terms in the expressions for perturbation coefficients A_1 and B_1 in (7).

⁶ Both the $\omega \rightarrow 0$ and $\omega \rightarrow 54$ limits give an infinitely strongly coupled theory. To avoid this, one can use, say, $1/2 < \omega < 53$.

are homogeneous and isotropic, they are such due to the diff invariance for $h_{\mu\nu}$, and Galilean invariance for either π or σ . The Galilean invariance, being exact in the DL, is only an approximate symmetry in the full theory. Therefore, the full theory solutions are likely to be only approximately homogeneous and isotropic; this is similar to pure massive gravity where such metrics approximate well the standard homogeneous and isotropic evolution [10], when the Vainshtein mechanism is at work⁷. Since our theory is that of special bi-Galileons, the Vainshtein mechanism is expected to work for a large domain of the parameter space. Moreover, the Higuchi problem need not arise as the Stüeckelberg fields are necessarily inhomogeneous/anisotropic. We will report on this in [22].

One may wonder whether quantum corrections can modify the classical picture given above - *e.g.* alter the viable parameter space given in Fig. 1. The answer is likely to be negative, due to the non-renormalization properties of Galileons [23], dRGT massive gravity [7], and related theories; these results can be summarized in the following statement: *None of the couplings in the Lagrangian (1) runs with energy below the scale Λ_3 in the decoupling limit.* Terms with at least two derivatives per field can certainly be generated via quantum loops, however they do not contribute to the equations of motion on the backgrounds considered here. These non-renormalization properties guarantee that any possible tuning of the parameters m , $a_{2,3}$ and ω , is technically natural [7].

Acknowledgements: We would like to thank Shinji Mukohyama and Enrico Trincherini for valuable communications. GG is supported by NSF and the NASA grant NNX12AF86G S06; R.K. is supported in part by a Grant-in-Aid for JSPS Fellows, and DP is supported in part by MIUR-FIRB grant RBFR12H1MW.

-
- [1] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. **A173**, 211 (1939).
 - [2] D. Boulware and S. Deser, Phys. Rev. **D6**, 3368 (1972).
 - [3] C. de Rham and G. Gabadadze, Phys. Rev. **D82**, 044020 (2010), arXiv:1007.0443 [hep-th].
 - [4] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys.Rev.Lett. **106**, 231101 (2011), arXiv:1011.1232 [hep-th].
 - [5] S. Hassan and R. A. Rosen, Phys.Rev.Lett. **108**, 041101 (2012), arXiv:1106.3344 [hep-th]; JHEP **1204**, 123 (2012), arXiv:1111.2070 [hep-th].
 - [6] M. Mirbabayi, Phys.Rev. **D86**, 084006 (2012), arXiv:1112.1435 [hep-th]; K. Hinterbichler and R. A.

- Rosen, JHEP **1207**, 047 (2012), arXiv:1203.5783 [hep-th]; C. Deffayet, J. Mourad, and G. Zahariade, JCAP **1301**, 032 (2013), arXiv:1207.6338 [hep-th]; T. Kugo and N. Ohta, (2014), arXiv:1401.3873 [hep-th].
- [7] C. de Rham, G. Gabadadze, L. Heisenberg, and D. Pirtskhalava, Phys.Rev. **D87**, 085017 (2013), arXiv:1212.4128; C. de Rham, L. Heisenberg, and R. H. Ribeiro, Phys.Rev. **D88**, 084058 (2013).
- [8] C. de Rham, G. Gabadadze, L. Heisenberg, and D. Pirtskhalava, Phys.Rev. **D83**, 103516 (2011), arXiv:1010.1780 [hep-th].
- [9] K. Koyama, G. Niz, and G. Tasinato, Phys.Rev.Lett. **107**, 131101 (2011), arXiv:1103.4708 [hep-th]; T. Nieuwenhuizen, Phys.Rev. **D84**, 024038 (2011).
- [10] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, *et al.*, Phys.Rev. **D84**, 124046 (2011), arXiv:1108.5231 [hep-th].
- [11] N. Khosravi, G. Niz, K. Koyama, and G. Tasinato, JCAP **1308**, 044 (2013), arXiv:1305.4950 [hep-th]; A. De Felice, A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, Class.Quant.Grav. **30**, 184004 (2013), arXiv:1304.0484 [hep-th].
- [12] G. D'Amico, G. Gabadadze, L. Hui, and D. Pirtskhalava, Phys.Rev. **D87**, 064037 (2013).
- [13] A. E. Gumrukcuoglu, K. Hinterbichler, C. Lin, S. Mukohyama, and M. Trodden, Phys.Rev. **D88**, 024023 (2013), arXiv:1304.0449 [hep-th].
- [14] G. D'Amico, G. Gabadadze, L. Hui, and D. Pirtskhalava, Class.Quant.Grav. **30**, 184005 (2013).
- [15] A. De Felice and S. Mukohyama, arXiv:1306.5502 [hep-th]; A. De Felice, A. E. Gumrukcuoglu, and S. Mukohyama, (2013).
- [16] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys.Rev. **D79**, 064036 (2009), arXiv:0811.2197 [hep-th].
- [17] A. Nicolis and R. Rattazzi, JHEP **0406**, 059 (2004), arXiv:hep-th/0404159 [hep-th]; A. Nicolis, R. Rattazzi, and E. Trincherini, JHEP **1005**, 095 (2010), arXiv:0912.4258 [hep-th].
- [18] L. Berezhiani, G. Chkareuli, C. de Rham, G. Gabadadze, and A. Tolley, Class.Quant.Grav. **30**, 184003 (2013), arXiv:1305.0271 [hep-th].
- [19] G. Gabadadze, K. Hinterbichler, D. Pirtskhalava, and Y. Shang, Phys.Rev. **D88**, 084003 (2013), arXiv:1307.2245 [hep-th].
- [20] N. A. Ondo and A. J. Tolley, JHEP **1311**, 059 (2013), arXiv:1307.4769 [hep-th].
- [21] A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, JCAP **1111**, 030 (2011), arXiv:1109.3845 [hep-th].
- [22] G. Gabadadze, R. Kimura, and D. Pirtskhalava, *In progress*.
- [23] M. A. Luty, M. Porrati, and R. Rattazzi, JHEP **0309**, 029 (2003); K. Hinterbichler, M. Trodden, and D. Wesley, Phys.Rev. **D82**, 124018 (2010).

⁷ Moreover, QMG solutions for the open universe may even be homogeneous and isotropic, as in pure massive gravity [21].